Eur. Phys. J. D **42**, 103–108 (2007)
DOI: 10.1140/epjd/e2006-00272-2

THE EUROPEAN PHYSICAL JOURNAL D

# Indirect decoherence in optical lattices and cold gases

D. Braun[a]

Laboratoire de Physique Théorique, IRSAMC, UMR 5152 du CNRS, Université Paul Sabatier, 118 route de Narbonne, 31062 Toulouse, France

Received 31 May 2006 / Received in final form 21 September 2006
Published online 15 December 2006 – 

**Abstract.** The interaction of two–level atoms with a common heat bath leads to an effective interaction between the atoms, such that with time the internal degrees of the atoms become correlated or even entangled. If part of the atoms remain unobserved this creates additional indirect decoherence for the selected atoms, on top of the direct decoherence due to the interaction with the heat bath. I show that indirect decoherence can drastically increase and even dominate the decoherence for sufficiently large times. I investigate indirect decoherence through thermal black body radiation quantitatively for atoms trapped at regular positions in an optical lattice as well as for atoms at random positions in a cold gas, and show how indirect decoherence can be controlled or even suppressed through experimentally accessible parameters.

**PACS.** 03.65.Yz Decoherence; open systems; quantum statistical methods – 03.67.Lx Quantum computation – 42.50.-p Quantum optics

With the rise of quantum information processing it has become necessary to understand decoherence in true many–particle systems. It has been known for a long time that the decoherence rate for a single degree of freedom scales like a power of a certain distance between the components of a superposition. The "distance" and its natural scale depend on the coupling to the heat bath. For example, if the single degree of freedom couples through a spatial coordinate $x$ to the heat bath, the latter selects eigenstates of $x$ as "pointer-basis" [1], and the relevant "distance" is measured in configuration space, with a microscopic length scale such as the thermal de Broglie length as natural unit. Decoherence therefore becomes extremely fast for mesoscopic or even macroscopic distances, and this is considered one of the main reasons why the everyday world around us behaves classically. Other couplings lead to different power laws and different natural microscopic units [2]. Decoherence processes for single degree of freedom systems are nowadays routinely resolved experimentally for microscopic distances between the superposed components, and have been so far in good agreement with the theoretical predictions [3–8]. However, decoherence measurements on true many particle systems are only now becoming available [9], and there is a need for detailed theoretical predictions, in order to verify the validity of quantum mechanics in an entirely new regime, namely one where the joined states of many particles are coherently superposed [10–12].

Theoretical progress was achieved recently with the derivation of a "decoherence metric" [13], which measures the distance between the components of a quantum superposition of arbitrarily many qubits with degenerate energy levels, and determines directly the time dependent decoherence. It turned out that for sufficiently far separated qubits with degenerate energy levels the time dependent decoherence boils down to just single qubit decoherence multiplied with the standard Hamming distance between the superposed quantum code words. For smaller qubit separations (the relevant length scale is the inverse of the wave length of the UV cut-off of the bath modes), interference effects start to play a role and one sees strong deviations from the simple scaling with the standard Hamming distance. Nevertheless, the notion of a distance (more precisely: a pseudo–metric, in the strict mathematical sense), can be maintained through the introduction of a metric tensor determined by the heat bath, whose off-diagonal elements reflect the interference processes. As a consequence, $2^{2N-1}$ independent decoherences are governed by only $\sim N^2$ matrix elements of the metric tensor. The entanglement of a state alone does not determine how fast it decoheres: for example, a GHZ state $(|000\rangle + |111\rangle)/\sqrt{2}$ has maximum Hamming distance between its two components, but if the qubits are sufficiently close the decoherence metric will distinguish this state for example from the state $(|001\rangle + |110\rangle)/\sqrt{2}$, or all other states which differ from GHZ by flipping qubits, whereas all theses states have the same entanglement.

The metric tensor contains the contribution from the direct decoherence process discussed so far, but also an

[a] e-mail: braun@irsamc.ups-tlse.fr

"indirect decoherence": the heat bath generates effective interactions between the qubits which can lead to classical correlations or even entanglement between them ("reservoir induced entanglement", [14]). "Indirect decoherence" is the additional decoherence that is induced if some of the atoms which got correlated or entangled with the selected atoms remain unobserved. In this paper, I investigate indirect decoherence in more detail, and show that even for a rather small number of unobserved atoms (of the order of 10), indirect decoherence can strongly enhance the overall decoherence. The effect should be important if one wants to build a quantum memory from trapped atoms in an optical lattice and if additional atoms get trapped in the optical lattice and are not read out. Indeed, until recently [15] it was difficult to even control the number of atoms per lattice site. The situation might be worse for quantum information stored in macroscopic gas samples [7,16,17], where the total number of atoms in which the information is stored can only be estimated and one cannot control which individual atoms store the quantum information.

Rydberg atoms have recently attracted particular attention as carrier of quantum information [16–18] due to the possibility of large dipole–dipole interactions, which have been studied in detail experimentally [19]. At the same time it is to be expected that strong interactions between the Rydberg atoms can also lead to strong indirect decoherence, and thus to possible experimental verification of the effect. We give numerical estimates of the time scales involved in indirect decoherence for the example of the dipole transition $46p_{3/2} \rightarrow 45d_{5/2}$ in a gas of cold Rydberg atoms studied in [19] and show that the effect is within experimental reach. Future research will have to determine if indirect decoherence also plays an essential role in other situations, notably schemes where the dominant coupling to the environment is *not* through the dipole operator (see e.g. [20–22]), or where indirect decoherence is much slower than the intrinsic time evolution of the system without decoherence.

## 1 The model

Let us consider $N$ two level atoms at arbitrary but fixed positions $\mathbf{R}_i$ ($i = 0, \dots N-1$) interacting with thermal black body radiation, which forms a common heat bath. All atoms are assumed identical with degenerate energy eigenstates $|-1\rangle$ and $|1\rangle$, with $\sigma_z|\pm 1\rangle = \pm|\pm 1\rangle$. In dipole coupling approximation, the total Hamiltonian reads [23]

$$H = \sum_k \hbar\omega_k a_k^\dagger a_k + \hbar \sum_k \sum_{i=0}^{N-1} g_k^{(i)} \sigma_{xi} \left( a_k \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{R}_i} + a_k^\dagger \mathrm{e}^{-\mathrm{i}\mathbf{k}\cdot\mathbf{R}_i} \right), \quad (1)$$

where $\sigma_{xi}$ and $\sigma_{zi}$ are Pauli matrices for atom $i$. The index $k$ stands for wave vector $\mathbf{k}$ and polarization direction $\lambda$ ($k_j = 2\pi n_j/L$ with integer $n_j$, $j = x, y, z$ for periodic boundary conditions); $a_k^\dagger$ ($a_k$) are the creation (annihilation) operators for mode $k$ with frequency $\omega_k = c|\mathbf{k}|$, polarization vector $\epsilon_k$, and electric field amplitude $\mathcal{E} = \sqrt{\hbar\omega_k/(2\varepsilon_0 V)}$, where $\varepsilon_0$, $c$, and $V$ are the dielectric constant of the vacuum, speed of light, and the quantization volume, respectively. The coupling constant of atom $i$ to mode $k$ is denoted by $g_k^{(i)} = -(ed\mathcal{E}/\hbar)\hat{u}^{(i)} \cdot \epsilon_k$, where $\hat{u}^{(i)}$ stands for a unit vector in the direction of the dipole moment of atom $i$, $\langle -1|\mathbf{d}|1\rangle = ed\hat{u}^{(i)}$ with electron charge $e$ and dipole length $d$. The restriction to atoms with degenerate energy levels, $\Omega_0 = 0$, leads to a vanishing system Hamiltonian, $H_{\mathrm{sys}} = (1/2)\hbar\Omega_0 \sum_{i=0}^{N-1} \sigma_{zi} = 0$. The model is a special case of the more general class of models known as pure dephasing models, where the system Hamiltonian commutes with the interaction Hamiltonian, i.e. the second term in (1). These models can be solved exactly for an arbitrary number of atoms at arbitrary positions. It is important to note that the results presented are robust in the sense that they remain unchanged for a small but finite level spacing $\Omega_0$. The system Hamiltonian is negligible if the fastest processes of the system itself are much slower than the decoherence times we are interested in [24]. We will see that such a regime appears for highly excited Rydberg atoms.

## 2 Decoherence metric

We are interested in the decoherence process of the $n$ selected atoms (indices $0, \dots, n-1$) out of the $N$ atoms. We therefore have to first trace out the electro–magnetic (e.m.) field modes, leaving a density matrix $\rho$, and secondly the unobserved atoms $n \dots N-1$. The resulting reduced density matrix $\tilde{\rho}(t)$ of the remaining atoms will be expressed in the eigenbasis of the $\sigma_{xi}$, $\sigma_{xi}|\pm 1\rangle_x = \pm|\pm 1\rangle_x$, the natural basis (also called pointer basis) for studying the decoherence process [1]. It has matrix elements $\tilde{\rho}_{\tilde{\mathbf{s}}\tilde{\mathbf{s}}'}(t) = \mathrm{tr}_{n\dots N-1}\rho_{\mathbf{s}\mathbf{s}'}(t)$, where $\tilde{\mathbf{s}}$ and $\tilde{\mathbf{s}}'$ are subsets of length $n$ of the labels $\mathbf{s} = (s_0, s_1, \dots s_{N-1})$ and $\mathbf{s}' = (s_0', s_1', \dots s_{N-1}')$ of the quantum states $|\mathbf{s}\rangle$ and $|\mathbf{s}'\rangle$ of all atoms, taken as column vectors, and $s_i, s_i' = \pm 1$, $i = 0, 1, \dots, N-1$, refer to atom $i$. We assume that all unobserved atoms are initially in the energy eigenstate $|1\rangle = (|1\rangle_x + |-1\rangle_x)/\sqrt{2}$, and that there are no initial correlations between the unobserved atoms and the selected atoms.

The dynamical quantities of interest are the "decoherences" $d_{\tilde{\mathbf{s}}\tilde{\mathbf{s}}'}(t)$, which we define as normalized complements of "coherences" (i.e. off-diagonal elements of the reduced density matrix of the $n$ selected atoms alone),

$$d_{\tilde{\mathbf{s}}\tilde{\mathbf{s}}'}(t) \equiv 1 - \frac{|\tilde{\rho}_{\tilde{\mathbf{s}}\tilde{\mathbf{s}}'}(t)|}{|\tilde{\rho}_{\tilde{\mathbf{s}}\tilde{\mathbf{s}}'}(0)|} \text{ for } \tilde{\rho}_{\tilde{\mathbf{s}}\tilde{\mathbf{s}}'}(0) \neq 0. \quad (2)$$

In [13] it was shown that the behavior of the decoherences is given by $d_{\tilde{\mathbf{s}}\tilde{\mathbf{s}}'}(t) \simeq ||\tilde{\mathbf{s}} - \tilde{\mathbf{s}}'||^2_{M(t)}$ with the "decoherence metric"

$$||\tilde{\mathbf{s}} - \tilde{\mathbf{s}}'||_{M(t)} \equiv \frac{1}{2}\sqrt{(\tilde{\mathbf{s}} - \tilde{\mathbf{s}}')^T \mathbf{M}(t) (\tilde{\mathbf{s}} - \tilde{\mathbf{s}}')}, \quad (3)$$

where $^T$ denotes the transpose, and $\mathbf{M}(t)$ is a real, symmetric, and non–negative time dependent "decoherence metric tensor" (DMT) with matrix elements ($i,j = 0,\ldots,n-1$, $\beta = 1/k_BT$ is the inverse temperature),

$$M_{ij}(t) = 4f_{ij}(t,\mathbf{R}_i-\mathbf{R}_j) + 2\Phi_{ij}(t,\mathbf{R}_i,\mathbf{R}_j), \quad (4)$$

$$f_{ij}(t,\mathbf{R}) = \sum_k \frac{g_k^{(i)}g_k^{(j)}}{\omega_k^2}\cos(\mathbf{k}\cdot\mathbf{R})(1-\cos\omega_k t) \times \coth\frac{\beta\hbar\omega_k}{2} \quad (5)$$

$$\varphi_{ij}(t,\mathbf{R}) = 2\sum_k \frac{g_k^{(i)}g_k^{(j)}}{\omega_k^2}\cos(\mathbf{k}\cdot\mathbf{R})(\omega_k t-\sin\omega_k t) \quad (6)$$

$$\Phi_{ij}(t,\mathbf{R}_i,\mathbf{R}_j) = \sum_{k=n}^{N-1}\varphi_{ik}(t,\mathbf{R}_i-\mathbf{R}_k)\varphi_{jk}(t,\mathbf{R}_j-\mathbf{R}_k). \quad (7)$$

The heat–bath itself therefore induces a natural distance $||\tilde{\mathbf{s}}-\tilde{\mathbf{s}}'||_{M(t)}$ between the $n$–qubit states, which determines directly the time dependent decoherences. The validity of equation (3) is limited to $|M_{ij}(t)| \ll 1\ \forall i,j$.

The distance $||\tilde{\mathbf{s}}-\tilde{\mathbf{s}}'||_{M(t)}$ generalizes the well–known Hamming distance $D^H(\tilde{\mathbf{s}},\tilde{\mathbf{s}}')$, which is defined as the number of bits in which $\tilde{\mathbf{s}}$ and $\tilde{\mathbf{s}}'$ differ, and which is obtained for $\mathbf{M} = \mathbf{I}$. This limit is reached for sufficiently large separation of the qubits, and $d_{\tilde{\mathbf{s}}\tilde{\mathbf{s}}'}(t)$ then goes over into $D^H(\tilde{\mathbf{s}},\tilde{\mathbf{s}}')$ up to a time dependent function describing single qubit decoherence [13].

It is clear from the definition that $\mathbf{M}(t)$ is real and symmetric. In Appendix A, I show that $\mathbf{M}(t)$ is also non–negative and obeys the triangle inequality. However, if a decoherence free subspace (DFS) [25–29] exists, the decoherence metric is strictly speaking a pseudo–metric, as there can be code words $\tilde{\mathbf{s}}$ and $\tilde{\mathbf{s}}'$ with $\tilde{\mathbf{s}} \neq \tilde{\mathbf{s}}'$ such that $||\tilde{\mathbf{s}}-\tilde{\mathbf{s}}'||_{M(t)} = 0$.

## 3 One selected atom

Indirect decoherence, i.e. the part $\Phi_{ij}$ in equation (4) is best appreciated for just $n = 1$ selected atom (taken to have index $i = 0$), and $N-1$ non–observed atoms. Only one decoherence is then relevant, $d_{1-1}$, and $\mathbf{M}$ has a single matrix element, $M_{00}(t) = 4f_{00}(t)+2\Phi_{00}(t)$. We express all lengths in terms of the dipole length $d$, $r_{ij} = |\mathbf{R}_i-\mathbf{R}_j|/d$, and times in units of $d/c$. We will furthermore assume that all dipoles are oriented in the same direction $\hat{u}$. The angle between $\hat{u}$ and the vector $\mathbf{r}_i-\mathbf{r}_j$ will be called $\theta_{ij}$, such that $\varphi_{ij}(t,\mathbf{R}_i-\mathbf{R}_j)$ becomes a function of $t$, $r_{ij}$, and $\theta_{ij}$, and depends on $i$ and $j$ only through these variables, $\varphi_{ij}(t,\mathbf{R}_i-\mathbf{R}_j) \equiv \varphi(t,r_{ij},\theta_{ij})$. As shown in [13], $f_{00}$ is given for $T = 0$ by

$$f_{00}(t) = \frac{2}{3\pi}\alpha\left(\frac{\kappa^2}{2}+\frac{1-\cos(\kappa t)-\kappa t\sin(\kappa t)}{t^2}\right), \quad (8)$$

where $\kappa = k_{\max}d$ is a UV cut–off of the heat bath. A necessary condition for the dipole-coupling approximation is $\kappa \ll 1$. Corrections to (8) due to finite temperature are of order $k_BT/(\hbar\omega_{\max})$ with $\omega_{\max} = ck_{\max}$, and will be neglected in the following. Equation (8) implies that for a finite UV–cutoff the direct decoherence will remain finite for all times,

$$f_{00} \stackrel{t\gg 1}{\longrightarrow} \frac{\alpha\kappa^2}{3\pi}. \quad (9)$$

Such a behavior has been termed "incomplete decoherence" [14]. The initial behavior is quadratic in $t$, $f_{00} \simeq \gamma^2t^2$ with $\gamma = \sqrt{\alpha/(12\pi)}\kappa^2$ for $\gamma t \ll 1$.

The function $\varphi(t,r,\theta)$ reads

$$\varphi(t,r,\theta) = \frac{2\alpha}{\pi r^2}\left\{\frac{1}{4}(1+3\cos(2\theta))\Big(\mathrm{Si}((r+t)\kappa)-\mathrm{Si}((r-t)\kappa)\Big) + \frac{3\sin^2\theta-2}{2r}\Big((r+t)\mathrm{Si}((r+t)\kappa) - (r-t)\mathrm{Si}((r-t)\kappa) - 2t\mathrm{Si}(\kappa r)\Big)\right\}, \quad (10)$$

where Si denotes the sin-integral. Rapidly oscillating terms of the type $\sin(\kappa r)$, $\cos(\kappa r)$, and $\kappa r\cos(\kappa r)$ have been neglected here, as their average in the case of a small uncertainty in the atom positions is exponentially small: atoms of mass $M$ trapped in the ground states of harmonic oscillators with trapping frequency $\nu$ have a Gaussian distribution of their center of mass with a width $\delta r \sim \sqrt{\hbar/\nu M}$, leading to a suppression of these terms by a factor $\exp(-(\kappa\delta r)^2/2)$. A typical experimental parameter, $\nu \sim 30$ kHz [30], leads to $\delta r d \sim 100$ nm. Optical dipole lengths $d \sim$Å and a UV cut-off $k_{\max} \sim 1/10$ Å give $\kappa \sim 0.1$, and $\kappa\delta r \sim 100$, so that these terms can indeed be safely neglected. For a cold gas $\delta r$ is expected to be even larger.

In the limit of $|r \pm t|\kappa \gg 1$, $\varphi_{ij}(t,r,\theta)$ approaches

$$\varphi(t,r,\theta) = \alpha\frac{t}{r^3}(3\cos^2\theta-1)\Theta(t/r-1), \quad (11)$$

where $\Theta(t/r-1)$ is the Heaviside function centered on the light cone. We therefore recognize $\varphi$ as a phase accumulated due to an effective dipole interaction between the atoms mediated through the modes of the electromagnetic field. Note that in this limit indirect decoherence becomes basically independent of the cut–off $\kappa$.

### 3.1 Optical lattices

In the following we consider specifically the situation for a 2D square optical lattice with lattice constant $a$ (taken in units of the dipole length $d$ as well), with the single selected atom at the center of the lattice. The fact that $\varphi(t,r,\theta) \propto t$ leads to an unbound quadratic growth of $\Phi_{ij}(t,\mathbf{R}_i,\mathbf{R}_j)$ with $t$, $\Phi_{ij} \sim N_{nn}(\alpha t/a^3)^2$, where $N_{nn}$ is an effective number of nearest neighbors, weighed by the inverse cube of their distance from the selected atom in units of the lattice spacing. As $\alpha\kappa^2 \ll 1$ an immediate

consequence of equations (11, 9) is that for large enough times, $t > t_1$ with

$$t_1 \sim \frac{\kappa a^3}{\sqrt{3\pi\alpha N_{nn}}}, \tag{12}$$

indirect decoherence always dominates over direct coherence even for a small number of unobserved atoms close to the selected atom. Due to the strong $r$-dependence of (11), the nearest neighbors and next nearest neighbors give the by far leading contributions to the indirect decoherence. Indirect decoherence dominates immediately (i.e. as soon as $t \gg a \gg 1$, where the quadratic behavior of $\Phi_{ij} \sim N_{nn}(\alpha t/a^3)^2$ in $t$ is valid) over direct decoherence, if $N_{nn}(\alpha t/a^3)^2 \gg \gamma^2 t^2$, or $a < a_c$ with the critical spacing $a_c \sim (12\pi\alpha N_{nn})^{1/6}/\kappa^{2/3}$. If the cut-off is of the order $\hbar\omega_{\max} \sim 1$ eV and $d = 1$ Å, $a_c d$ is of the order 100 nm. If the cut-off is given by the break–down of the dipole approximation ($k_{\max} \sim 2\pi/d$), $a_c$ reduces to $a_c \sim 1$, i.e. the atoms will have to become basically closely packed before indirect decoherence immediately dominates over direct decoherence.

One might object that a small number of two–level atoms with which the selected atoms effectively interact cannot constitute a real heat-bath, and should rather lead to repeating revival phenomena of the coherences instead of to decoherence. However, it turns out that even for a square optical lattice of $3 \times 3$ atoms (i.e. 8 unobserved atoms), the revivals are hardly visible, and in a square optical lattice of $31 \times 31$ atoms all revivals seem to have disappeared completely. This is shown in Figure 1 where we see the decoherence as function of time for a 2D square optical lattice with lattice constant $a = 1000$, dipole moments perpendicular to the plane of the lattice, and with the selected atom in the center of the lattice. Direct decoherence sets in immediately and increases $\propto t^2$ for small times (see Eq. (8)), before saturating at $\alpha\kappa^2/(3\pi)$. Due to the weakness of the dipole coupling, indirect decoherence becomes appreciable only at much later times for atoms separated thus far. But because of the continued quadratic growth of the indirect decoherence it finally destroys all coherence left by the direct decoherence. The figure also shows that the exact result for $d_{1-1}$ [13] is very well approximated in the entire interesting regime $d_{1-1} \leq 1$ by the decoherence metric prediction, equation (3). Indirect decoherence has an interesting dependence on the orientation of the dipoles. Equation (11) shows that $\Phi_{00} \propto (3\cos^2\theta - 1)^2$. Indirect decoherence in a 2D optical lattice can therefore be completely suppressed by orienting the atomic dipoles at the magical angle $\theta = \arccos(1/\sqrt{3}) \simeq 54.7°$ with respect to the lattice. This angular dependence might serve as additional experimental signature of the effect.

Let us consider as specific physical example highly excited Rydberg atoms. In [19] the transition $46p_{3/2} \rightarrow 45d_{5/2}$ in Rydberg atoms was studied experimentally. These two levels have an allowed dipole transition with $d \sim 2600$ in atomic units, and a level spacing $\Omega_0 \sim 10$ GHz. If we consider a lattice constant $a \sim 10$ (in units of $d$, corresponding to about 1 $\mu$m, roughly a factor 100 smaller than the average spacing in the cold gas considered in [19]), indirect decoherence arises on the time scale $\tau_{\rm dec} \sim (1/\sqrt{N_{nn}})a^3/\alpha$, which scales like $1/d^2$, and gives a numerical value of a fraction of $10^{-10}$ s, i.e. shorter than the inverse level spacing. Going to even higher levels or higher densities leads to even stronger separation of the two time scales, as the dipole matrix element roughly increases like $n^2$ with the main quantum number $n$, and the level spacing decays as $1/n^3$. Thus, for such highly excited and relatively densely packed Rydberg atoms, the model can be applied, and predicts indirect decoherence on a sub-ns time scale, faster than the intrinsic time evolution of single Rydberg atom put into superposition of the two states $46p_{3/2}$ and $45d_{5/2}$.

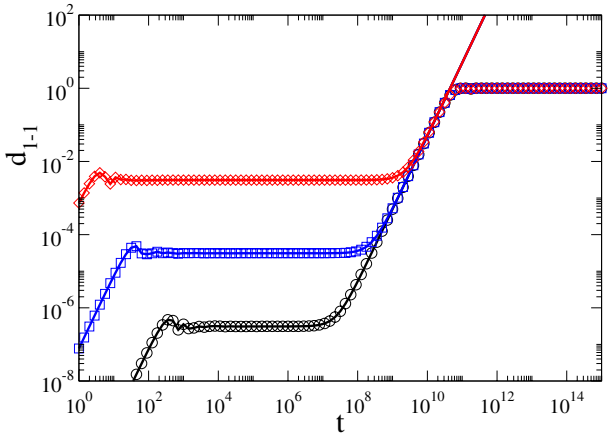


**Fig. 1.** (Color online) The decoherence $d_{1-1}(t)$ for a single selected atom in the center of a 2D square optical lattice of $31 \times 31$ atoms with lattice constant $a = 1000$ (in units of the dipole length) as a function of the dimensionless time $t$ for $\kappa = 0.01$ (black circles), $\kappa = 0.1$ (blue squares), and $\kappa = 1$ (red diamonds) along with the decoherence metric predictions (continuous lines). The first rise corresponds to the contribution of direct decoherence due to the interaction with the e.m. modes, the second rise results from indirect decoherence due to the effective interaction with the non-selected atoms mediated by the e.m. modes. Superpositions are decohered completely when $d_{1-1} = 1$ is reached.

### 3.2 Cold gases

For an atomic gas, the positions $\mathbf{R}_j$ of the atoms are not known. We resort to an ensemble description, where we average over the positions of the atoms. We assume that all $\mathbf{R}_j$ with the exception of $\mathbf{R}_0 = \mathbf{0}$ are randomly, independently and evenly distributed with an average density (atoms per volume) $\rho_V$. We find

$$\begin{aligned}\langle\Phi_{00}(t,\mathbf{0},\mathbf{0})\rangle &= \left\langle \sum_{k=1}^{N-1} \varphi_{0k}^2(t,\mathbf{R}_k) \right\rangle \\ &= 2\pi\rho_V d^3 \int_l dr\, r^2 \int_0^\pi d\vartheta \sin\vartheta (\alpha t/r^3)^2 \\ &\quad \times (3\cos^2\vartheta - 1)^2 \Theta(t/r - 1). \end{aligned} \tag{13}$$

The lower cut-off is now given by the smallest distance up to which two atoms might approach each other, which for a sufficiently dilute gas at low temperature is of the order of the scattering length $l$ (taken in units of $d$ as well), if $l > 0$. This gives

$$\langle \Phi_{00}(t,0,0)\rangle \simeq \gamma_G^2 t^2 \quad \text{with} \quad \gamma_G = \alpha\sqrt{\frac{16\pi d^3\rho_V}{15 l^3}}. \qquad (14)$$

Indirect decoherence immediately dominates over direct decoherence for densities $\rho_V \gtrsim \kappa^4 l^3/d^3$. A UV cut-off $\hbar\omega_{\max} = 1$ eV, $d = 1$ Å, $l = 10$ Å gives a critical density of about $10^{20}$ atoms/m$^3$. Recent experiments on dense Bose-Einstein condensates deal already with similar densities [31]. The coherence of internal (spin) degrees of freedom of condensed bosons has recently been demonstrated [32], so that indirect decoherence in a cold gas might become observable in the near future. For smaller densities indirect decoherence takes over for $t > t_2$ with

$$t_2 \sim \kappa\sqrt{\frac{l^3}{\rho_V d^3}}. \qquad (15)$$

The dependence of the indirect decoherence on $l$ gives the interesting perspective to control indirect decoherence in a cold gas through a Feshbach resonance, which allows to vary $l$ over many orders of magnitude [33].

## 4 Conclusions

I have shown that indirect decoherence due to reservoir induced entanglement between degenerate two–level atoms can substantially increase decoherence in an optical lattice or a cold atomic gas, compared to the direct decoherence due to the coupling of each atom to the e.m. field. For large enough times indirect decoherence in fact always dominates, even for only a few unobserved atoms. For sufficiently densely packed atoms the dominance of indirect decoherence begins as soon as a light signal has traveled a dipole length. The dependence of the indirect decoherence on the orientation of and the distance between the dipoles offers the interesting perspective to control indirect decoherence with easily accessible parameters. In a 2D optical lattice, indirect decoherence can be switched off completely by orienting all dipoles under a magical angle $\theta = \arccos(1/\sqrt{3})$ with respect to the lattice, and in a dilute, cold atomic gas, one can suppress indirect decoherence to large extent by increasing the scattering length $l$ through a Feshbach resonance. The effect presented here should be observable experimentally with highly excited Rydberg atoms.

I wish to thank Olivier Giraud for useful discussions. This work was supported in part by the Agence National de la Recherche (ANR), project INFOSYSQQ, and EC IST-FET projects EDIQIP and EuroSQIP.

## Appendix A: Proof of non–negativity and of the triangle inequality for the decoherence metric

### A.1 Non–negativity

We show separately $\sum_i x_i f_{ij} x_j \geq 0$ and $\sum_i x_i \Phi_{ij} x_j \geq 0$ $\forall x_i \in \mathbb{R}$. From equations (5, 6) we have

$$\begin{aligned}\sum_{i,j} x_i f_{ij} x_j &= \sum_{i,j}\sum_k x_i x_j g_k^{(i)} g_k^{(j)} \mathrm{e}^{i\mathbf{k}\cdot(\mathbf{R}_i-\mathbf{R}_j)} \frac{1-\cos\omega_k t}{\omega_k^2} \\ &\quad \times \coth\frac{\beta\hbar\omega_k}{2} \\ &= \sum_k \left|\sum_i x_i g_k^{(i)} \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot R_i}\right|^2 \frac{1-\cos\omega_k t}{\omega_k^2} \\ &\quad \times \coth\frac{\beta\hbar\omega_k}{2} \geq 0\end{aligned}$$

$$\sum_{i,j} x_i \Phi_{ij} x_j = \sum_{i,j}\sum_l x_i \varphi_{il}\varphi_{jl} x_j = \sum_l \left(\sum_i x_i \varphi_{il}\right)^2 \geq 0.$$

Thus, also $\sum_{i,j} x_i M_{ij} x_j \geq 0$, and $\mathbf{M}$ is therefore non–negative.

### A.2 Triangle inequality

We define the linear map $\phi$: $\mathbb{R}^n \to \mathbb{R}^n$, $\mathbf{v} \to \mathbf{M}\mathbf{v}$, where $\mathbf{M}$ is a real symmetric, non–negative $n \times n$ matrix, i.e. $\mathbf{v}^T\mathbf{M}\mathbf{v} \geq 0$ for all $\mathbf{v} \in \mathbb{R}^n$. We also define the bilinear form $(\cdot,\cdot)$: $\mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$, $(\mathbf{v},\mathbf{w}) = \mathbf{v}^T\mathbf{M}\mathbf{w}$, which is not a scalar product, as $(\mathbf{v},\mathbf{v})$ can be zero for $\mathbf{v} \neq 0$. One can nevertheless prove the Cauchy—Schwartz (C.S.) inequality $(\mathbf{v},\mathbf{w})^2 \leq (\mathbf{v},\mathbf{v})(\mathbf{w},\mathbf{w})$.

Let $V_0$ be the kernel of $\phi$. Thus $\mathbf{M}\mathbf{v} = 0 = \mathbf{v}^T\mathbf{M}$ $\forall \mathbf{v} \in V_0$. Suppose first that $\mathbf{v} \in V_0$ or $\mathbf{w} \in V_0$. Then $(\mathbf{v},\mathbf{w}) = 0$, but also $(\mathbf{v},\mathbf{v})(\mathbf{w},\mathbf{w}) = 0$, as at least one factor is zero. Thus the C.S. inequality is trivially fulfilled. Now suppose that $\mathbf{v} \notin V_0$ and $\mathbf{w} \notin V_0$. Define $\tilde{\mathbf{v}} = (\mathbf{w},\mathbf{w})\mathbf{v} - (\mathbf{w},\mathbf{v})\mathbf{w}$. We have

$$0 \leq (\tilde{\mathbf{v}},\tilde{\mathbf{v}}) = (\mathbf{w},\mathbf{w})[(\mathbf{w},\mathbf{w})(\mathbf{v},\mathbf{v}) - (\mathbf{w},\mathbf{v})^2]. \qquad (16)$$

It is easily seen that $(\mathbf{w},\mathbf{w}) \neq 0$ if $\mathbf{w} \notin V_0$: decompose $\mathbf{M} = \mathbf{M}_0 + \mathbf{M}_+$ with $\mathbf{M}_0 = P_0\mathbf{M}P_0$, $\mathbf{M}_+ = (1-P_0)\mathbf{M}(1-P_0)$, where $P_0$ is the projector onto $V_0$. $\mathbf{M}_+$ is the positive part of the map, i.e. $\mathbf{w}^T\mathbf{M}_+\mathbf{w} > 0$ $\forall \mathbf{w} \neq 0$. Thus, from $(\mathbf{w},\mathbf{w}) = 0$ follows $\mathbf{w} = 0$ or $\mathbf{M}_+ = 0$. In both cases $\mathbf{w} \in V_0$. Thus, for $\mathbf{w} \notin V_0$ we have $(\mathbf{w},\mathbf{w}) \neq 0$, and as $\mathbf{M} \geq 0$, this means $(\mathbf{w},\mathbf{w}) > 0$. Therefore equation (16) immediately gives the C.S. inequality.

The proof of the triangle inequality $||\tilde{\mathbf{s}} - \tilde{\mathbf{s}}''||_M \leq ||\tilde{\mathbf{s}} - \tilde{\mathbf{s}}'||_M + ||\tilde{\mathbf{s}}' - \tilde{\mathbf{s}}''||_M$ then proceeds in the usual fashion. One defines the norm $||\mathbf{x}||_M = \sqrt{(\mathbf{x},\mathbf{x})}$, and the C.S. inequality gives $||\mathbf{x}+\mathbf{y}||_M^2 = (\mathbf{x},\mathbf{x}) + (\mathbf{y},\mathbf{y}) + 2(\mathbf{x},\mathbf{y}) \leq (\mathbf{x},\mathbf{x}) + (\mathbf{y},\mathbf{y}) + 2|(\mathbf{x},\mathbf{y})| \leq ||\mathbf{x}||_M^2 + ||\mathbf{y}||_M^2 + 2||\mathbf{x}||_M\,||\mathbf{y}||_M = (||\mathbf{x}||_M + ||\mathbf{y}||_M)^2$. The triangle inequality for the distance (3) follows from here by setting $\mathbf{x} = \tilde{\mathbf{s}} - \tilde{\mathbf{s}}'$, $\mathbf{y} = \tilde{\mathbf{s}}' - \tilde{\mathbf{s}}''$.

## References


1. W.H. Zurek, Phys. Rev. D **24**, 1516 (1981)
2. W.T. Strunz, F. Haake, D. Braun, Phys. Rev. A **67**, 022101 (2003)
3. M. Brune, E. Hagley, J. Dreyer, X. Maître, A. Maali, C. Wunderlich, J.M. Raimond, S. Haroche, Phys. Rev. Lett. **77**, 4887 (1996)
4. D. Vion, A. Aassime, A. Cottet, P. Joyez, H. Pothier, C. Urbina, D. Esteve, M. Devoret, Science **296**, 886 (2002)
5. Y. Yu, S. Han, X. Chu, S.-I. Chu, Z. Wang, Science **296**, 889 (2002)
6. K. Hornberger, S. Uttenthaler, B. Brezger, L. Hackermüller, M. Arndt, A. Zeilinger, Phys. Rev. Lett. **90**, 160401 (2003)
7. B. Juulsgaard, J. Sherson, J. Fiurášek, E.S. Polzik, Nature **432**, 482 (2004)
8. C. Langer et al., Phys. Rev. Lett. **95**, 060502 (2005)
9. H.G. Krojanski, D. Suter, Phys. Rev. Lett. **93**, 090501 (2004)
10. W. Dür, C. Simon, J.I. Cirac, Phys. Rev. Lett. **89**, 210402 (2002)
11. F. Mintert, A.R.R. Carvalho, M. Kus, A. Buchleitner, Phys. Rep. **415**, 207 (2005)
12. A.J. Leggett, J. Phys.: Condens. Matter **14**, R415 (2002)
13. D. Braun, Phys. Rev. Lett. **96**, 230502 (2006)
14. D. Braun, Phys. Rev. Lett. **89**, 277901 (2002)
15. I. Bloch, Nat. Phys. **1**, 23 (2005)
16. D. Jaksch, J.I. Cirac, P. Zoller, S.L. Rolston, R. Côté, M.D. Lukin, Phys. Rev. Lett. **85**, 2208 (2000)
17. M.D. Lukin, M. Fleischhauer, R. Cote, L.M. Duan, D. Jaksch, J.I. Cirac, P. Zoller, Phys. Rev. Lett. **87**, 037901 (2001)
18. J.M. Raimond, M. Brune, S. Haroche, Rev. Mod. Phys. **73**, 565 (2001)
19. K. Afrousheh, P. Bohlouli-Sanjani, D. Vagale, A. Mugford, M. Fedorov, J. Martin, Phys. Rev. Lett. **93**, 233001 (2004)
20. D. Leibfried et al., Nature **438**, 639 (2005)
21. H. Häffner et al., Nature **438**, 643 (2005)
22. B.B. Blinov, D.L. Moehring, L.-M. Duan, C. Monroe, Nature **428**, 153 (2004)
23. M. Scully, M. Zubairy, *Quantum Optics* (Cambridge University Press, Cambridge, UK, 1997)
24. D. Braun, F. Haake, W. Strunz, Phys. Rev. Lett. **86**, 2913 (2001)
25. P. Zanardi, M. Rasetti, Phys. Rev. Lett. **79**, 3306 (1997)
26. L.M. Duan, G.C. Guo, Phys. Rev. A **57**, 2399 (1998)
27. D.A. Lidar, I.L. Chuang, K.B. Whaley, Phys. Rev. Lett. **81**, 2594 (1998)
28. D. Braun, P.A. Braun, F. Haake, *Proceedings of the 1998 Bielefeld Conference on "Decoherence: Theoretical, Experimental, and Conceptual Problems"*, Lect. Notes Phys. **538**, 55 (2000)
29. J.B. Altepeter, P.G. Hadley, S.M. Wendelken, A.J. Berglund, P.G. Kwiat, Phys. Rev. Lett. **92**, 147901 (2004)
30. M. Greiner, O. Mandel, T. Esslinger, T.W. Hänsch, I. Bloch, Nature **415**, 39 (2002)
31. J. Schuster, A. Marte, S. Amtage, B. Sang, G. Rempe, H.C.W. Beijerinck, Phys. Rev. Lett. **87**, 170404 (2001)
32. M.-S. Chang, Q. Qin, W. Zhang, L. You, M.S. Chapman, Nat. Phys. **1**, 111 (2005)
33. H. Feshbach, Ann. Phys. (NY) **5**, 357 (1958)